Calculus AB
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For 2 - 4) Give the positions $s = f(t)$ of a body moving on a coordinate line, with s in meters and t in seconds.

(a) Find the body's displacement and average velocity for the given time interval.
(b) Fine the body's speed and acceleration at the endpoints of the interval.
(c) When if ever during the time interval does the body change direction?

2) $s - 6t - t^2$, [0,6]

3) $s = -t^3 + 3t^2 - 3t$, [0,3]

4) $s = \frac{t^4}{4} - t^3 + t^2$, [0,3]

5) At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ m.

a) Find the body's acceleration each time the velocity is zero.
b) Find the body's speed each time the acceleration is zero.
c) Find the total distance traveled by the body from $t = 0$ to $t = 2$.

6) At time $t \geq 0$, the velocity of a body moving along the s-axis is $v = t^2 - 4t + 3$.
   a) Find the body's acceleration each time the velocity is zero.
   b) When is the body moving forward? Backward?
   c) When is the body's velocity increasing? Decreasing?

7) The equations for free fall at the surfaces of Mars and Jupiter (s in meters, t in seconds) are $s = 1.86t^2$ on Mars and $s = 11.44t^2$ on Jupiter. How long does it take a rock falling from rest to reach a velocity of 27.8 m/sec (about 100 km/hr) on each planet?

8) A rock thrown vertically upward from the surface of the moon at velocity of 24 m/sec (about 86 km/hr) reaches a height of $s = 24t - 0.8t^2$ meters in t sec.
   a) Find the rock's velocity and acceleration at time$t$.
   b) How long does it take the rock to reach its highest point?
   c) How high does it go?
   d) How long does it take the rock to reach half its maximum height?
   e) How long is the rock aloft?

10) A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s = 832t - 2.6t^2$ feet after t sec. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ ft after t sec. How long will the bullet be aloft in each case? How high will the bullet go?

11) Had Galileo dropped a cannonball from the Tower of Pisa, 179 ft above the ground, the ball's height above ground t sec into the fall would have been $s = 179 - 16t^2$.

a) What would have been the ball's velocity, speed, and acceleration at time t?
b) About how long would it have taken the ball to hit the ground?
c) What would have been the ball's velocity at the moment of impact?

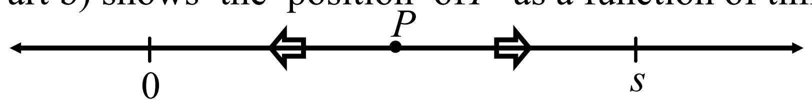
12) Galileo developed a formula for a body's velocity during free fall by rolling balls from rest down increasingly steep inclined planks and looking for a limiting formula that would predict a ball's behavior when the plank was vertical and the ball fell freely. He found that, for any given angle of the plank, the ball's velocity t sec into motion was a constant multiple of t. That is, the velocity was given by a formula of the form $v = kt$. The value of the constant k depended on the inclination of the plank.

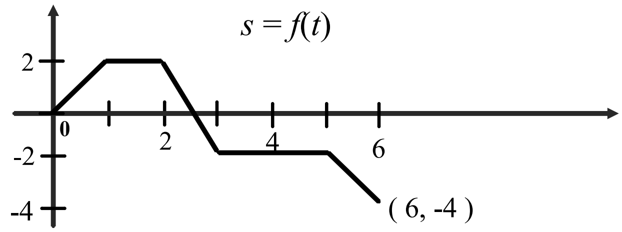
In modern notation - with distance in meters and time in seconds, what Galileo determined by experiment was that, for any given angle θ, the ball's velocity t sec into the roll was $v = 9.8(\sin\theta)t$ m/sec.

a) What is the equation for the ball's velocity during free fall?

b) What constant acceleration does a freely falling body experience near the surface of the Earth?

14) A particle P moves on the number line shown in part a) of the accompanying figure. Part b) shows the position of P as a function of time t.



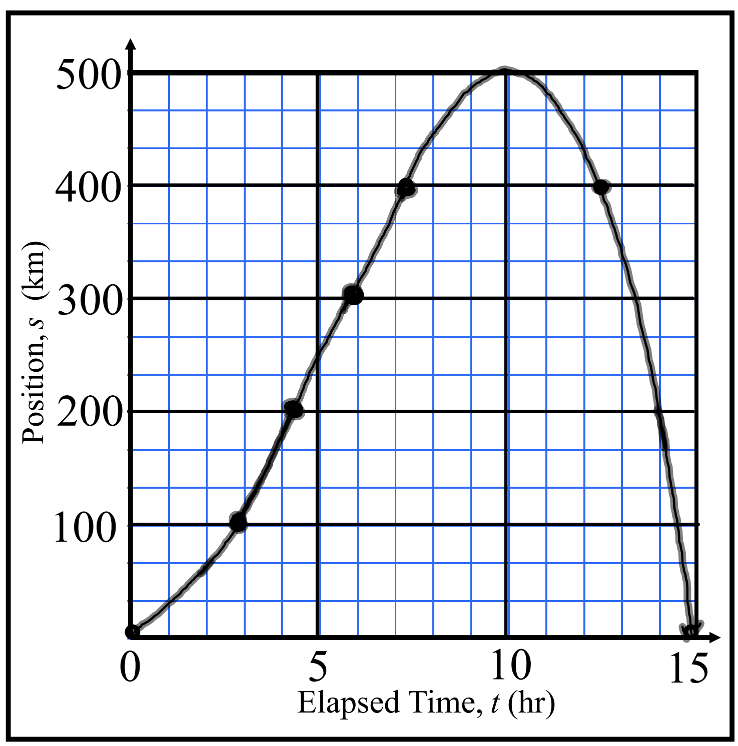


a) When is P moving to the left? Moving to the right? Standing still?

b) Graph the particle's velocity and speed (where defined).

16) The accompanying graph shows the position of s of a truck traveling on a highway. The truck starts at $t = 0$ and returns 15 h later at $t = 15$.

a) Graph the trucks velocity $v = \frac{ds}{dt}$ for $0 \leq t \leq 15$. Then repeat the process with the velocity curve, to graph the trucks acceleration $\frac{dv}{dt}$.



b) Suppose that

$$s = 15t^2 - t^3.$$

Graph $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$ and compare your graphs with those in part a)

22) When a bactericide was added to a nutrient broth in which bacteria were growing, the bacterium population continued to grow for a while, but then stopped growing and began to decline. The size of the population at time t (hours) was $b = 10^6 + 10^4 t - 10^3 t^2$.

Find the growth rates at:

a) $t = 0$ h

b) $t = 5$ h

c) $t = 10$ h

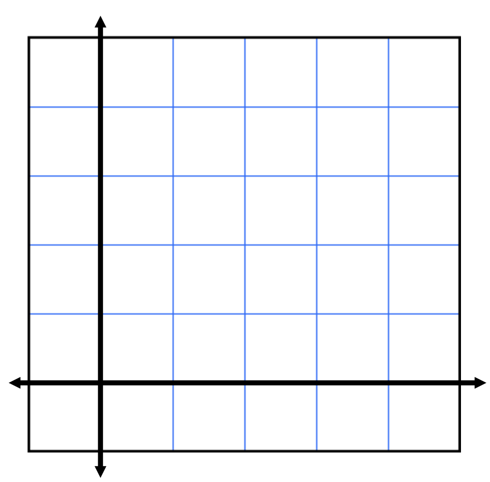
23) The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$. How fast is the water running out at the end of 10 min? What is the average rate at which the water flows out during the first 10 min?

24) It takes 12 h to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank after the valve is opened is given by the formula $y = 6\left(1 - \frac{t}{12}\right)^2$ meters.

a) Find the rate $\frac{dy}{dt}$ (m/hr) at which the tank is draining at time t.

b) When is the fluid level in the tank falling fastest? Slowest? What are the values of $\frac{dy}{dt}$ at these times?

c) Graph y and $\frac{dy}{dt}$ together and discuss the behavior of y in relation to the signs and values of $\frac{dy}{dt}$.



25) The volume of $V = \frac{4}{3}\pi r^3$ of a spherical balloon changes with the radius.

a) At what rate (ft^3/ft) does the volume change with respect to the radius when $r = 2$ ft?

b) By approximately how much does the volume increase when the radius changes from 2 to 2.2 ft?